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A Comparative Crystallographical Study of the Double Selenates of the Series $R_2M(SeO_4)_2,6H_2O$. Part II. Salts in Which M is Magnesium

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VII. *A Comparative Crystallographical Study of the Double Selenates of the Series* $R_2M(SeO_4)_2,6H_2O$.—*Part* II. *Salts in which* M *is Magnesium.*

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[Plate 9.]

In this memoir are communicated the results of an investigation of the magnesium group of double selenates, in which R is represented by potassium, rubidium, and cæsium. It is analogous and strictly comparable to that concerning the salts of the zinc group, which was presented to the Royal Society in March, 1900 ('Roy. Soc. Proc.,' vol. 67, p. 58). The potassium salt of the series was partially studied by Topsöe and Christiansen,* in the year 1874, with the less perfect means then available; the rubidium and cæsium salts have not hitherto been investigated.

The three salts were prepared precisely as in the case of the zinc salts, except that the normal magnesium selenate required for the production of the double salt, by addition to the calculated amount of the normal alkaline selenate, was prepared by digestion of the calculated quantity of pure diluted selenic acid with excess of pure calcined magnesia and subsequent filtration.

The same spherical projection is common to the series of double selenates and that of the double sulphates previously investigated by the author, and is given on p. 343 of the memoir describing the latter.†

Potassium Magnesium Selenate, $K_2Mg(SeO_4)_2,6H_2O$.

A determination of the content of magnesium in a specimen of the crystals employed afforded the following result:—1·1430 gramme yielded 0·2623 gramme of magnesium pyrophosphate, corresponding to 4·95 per cent. of magnesium. The calculated percentage of Mg is 4·84.

Goniometry.

Twelve good crystals of small size, selected out of five crops, were used in the measurements.

* 'Ann. de Chim. et de Phys.,' vol. 1, 1874, p. 75.

† 'Journ. Chem. Soc., Trans.,' 1893.

Habit: Short prismatic to tabular.

Axial angle: $\beta = 75° 42'$.

Ratio of axes: $a : b : c = 0·7485 : 1 : 0·5031$.

Forms observed: $a = \{100\} \infty \bar{P} \infty$; $b = \{010\} \infty \grave{P} \infty$; $c = \{001\} oP$; $p = \{110\} \infty P$; $q = \{011\} \grave{P} \infty$; $r' = \{\bar{2}01\} + 2\bar{P} \infty$; $o' = \{\bar{1}11\} + P$.

The results of the measurements are set forth in the accompanying table of angles.

The crystals were distinguished by the preponderating development of the c and p faces. Faces of the form r' were generally present, but small, while the q faces were even smaller and frequently absent altogether. A typical crystal is shown later in fig. 1 (p. 274), in the discussion of the comparative habits of the three magnesium salts. Many of the crystals resembled fig. 21 in the double sulphate memoir (*loc. cit.*, p. 384), except that there was no trace of b or o' faces. It occasionally happened that while one r' face was very small, the other one was relatively largely developed. The orthopinacoid a was discovered fairly well formed on one of the crops investigated, but was absent on the other crops. A trace of the hemi-pyramid o' was observed on one crystal, but the signal images afforded were not adequately good for the purposes of measurement.

Morphological Angles of Potassium Magnesium Selenate.

Angle measured.	No. of measurements.	Limits.	Mean observed.	Calculated.	Difference.
		° ′ ° ′	° ′	° ′	′
ac = 100 : 001 = β	—	—	—	75 42	—
as = 100 : 101	—	—	—	46 30	—
sc = 101 : 001	—	—	—	29 12	—
cr' = 001 : $\bar{2}$01	21	62 55— 63 4	62 58	62 53	5
cs' = 001 : $\bar{1}$01	—	—	—	38 1	—
$s'r'$ = $\bar{1}$01 : $\bar{2}$01	—	—	—	24 52	—
$r'a$ = $\bar{2}$01 : $\bar{1}$00	—	—	—	41 25	—
$r'c$ = $\bar{2}$01 : 00$\bar{1}$	21	116 57—117 8	117 2	117 7	5
ap = 100 : 110	2	35 53— 35 55	35 54	35 55	1
pp' = 110 : 120	—	—	—	19 28	—
$p'b$ = 120 : 010	—	—	—	34 37	—
pb = 110 : 010	2	54 5— 54 7	54 6	54 5	1
pp = 110 : 1$\bar{1}$0	17	71 42— 71 57	71 50	*	—
pp = 110 : $\bar{1}$10	18	108 0—108 21	108 10	108 10	0
cq = 001 : 011	28	25 53— 26 8	26 0	*	—
qb = 011 : 010	3	64 0— 64 6	64 2	64 0	2
qq = 011 : 01$\bar{1}$	6	128 0—128 3	128 1	128 0	1
ao = 100 : 111	—	—	—	49 43	—
oq = 111 : 011	—	—	—	27 27	—
aq = 100 : 011	—	—	—	77 10	—
qo' = 011 : $\bar{1}$11	—	—	—	34 16	—
$o'a$ = $\bar{1}$11 : $\bar{1}$00	—	—	—	68 34	—

Morphological Angles of Potassium Magnesium Selenate—*continued*.

Angle measured.	No. of measurements.	Limits.	Mean observed.	Calculated.	Difference.
		° ′ ° ′	° ′	° ′	′
co = 001 : 111	—	—	—	34 54	—
op = 111 : 110	—	—	—	43 34	—
cp = 001 : 110	48	78 20— 78 34	78 28	*	—
po′ = 110 : 11$\bar{1}$	—	—	—	57 13	—
o′c = 11$\bar{1}$: 00$\bar{1}$	—	—	—	44 19	—
pc = 110 : 00$\bar{1}$	48	101 22—101 42	101 32	101 32	0
bo = 010 : 111	—	—	—	69 57	—
os = 111 : 101	—	—	—	20 3	—
bo′ = 010 : $\bar{1}$11	—	—	—	65 16	—
o′s′ = $\bar{1}$11 : $\bar{1}$01	—	—	—	24 44	—
sq = 101 : 011	—	—	—	38 19	—
qp = 011 : $\bar{1}$10	13	85 26— 85 46	85 35	85 34	1
ps = $\bar{1}$10 : $\bar{1}$0$\bar{1}$	—	—	—	56 7	—
pq = $\bar{1}$10 : 0$\bar{1}\bar{1}$	13	94 12— 94 40	94 25	94 26	1
s′q = $\bar{1}$01 : 011	—	—	—	44 55	—
qp = 011 : 110	14	64 0— 64 15	64 7	64 5	2
ps′ = 110 : 10$\bar{1}$	—	—	—	71 0	—
pq = 110 : 0$\bar{1}\bar{1}$	14	115 45—116 5	115 53	115 55	2
r′o′ = $\bar{2}$01 : $\bar{1}$11	—	—	—	34 30	—
o′p = $\bar{1}$11 : 110	—	—	—	92 54	—
pr′ = 110 : 20$\bar{1}$	28	52 21— 52 42	52 33	52 36	3
r′p = $\bar{2}$01 : 110	28	127 18—127 37	127 27	127 24	3

Total number of measurements, 324.

From measurements made by Topsöe in 1870, Topsöe and Christiansen (*loc. cit.*, p. 75) quote for the ratio of the axes and the axial angle $a : b : c = 0{\cdot}7447 : 1 : 0{\cdot}5014$, and $\beta = 75° \ 43'$.

The cleavage direction common to the series, namely, parallel to r' $\{\bar{2}01\}$, is well developed.

Volume.

Relative density.—The following four determinations were carried out with different specimens of the salt :—

Weight of salt employed.	Sp. gr. at 20°/4°.	
7·4818	2·3627	Mean, 2·3630
6·2738	2·3630	
7·1272	2·3634	
6·2631	2·3630	

Molecular Volume.—$M/d = 495 \cdot 6 \div 2 \cdot 363 = 209 \cdot 73$.

Distance Ratios.—The molecular volume combined with the axial ratios and axial angle yield the following distance ratios :—

$$\chi : \psi : \omega = 6 \cdot 2233 : 8 \cdot 3144 : 4 \cdot 1829.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The symmetry plane is the plane of the optic axes (binormals). The sign of the double refraction is positive.

The following extinction angles were afforded by two section plates ground parallel to the symmetry plane. They are referred to the normal to the basal plane :—

Section 1, 3° 40′ ; section 2, 2° 20′ ; Mean, 3° 0′.

TOPSÖE and CHRISTIANSEN give 2° 0′ as the mean of two determinations.

The direction is behind the normal (that is, nearer to the vertical axis *c*). This direction is the second median line, the first median line being consequently also in the obtuse angle *ac* of the morphological axes and inclined 3° 0′ to the axis *a*. The second median line is inclined 11° 18′ to the vertical axis *c*.

Refractive Indices.—The six prisms employed were ground on six different crystals. The results are given in the accompanying table.

Refractive Indices of Potassium Magnesium Selenate.

	Light.	Prism 1.	Prism 2.	Prism 3.	Prism 4.	Prism 5.	Prism 6.	Mean.	Values of TOPSÖE and CHRISTIANSEN.
α Vibrations parallel 2nd median line	Li	—	1·4933	—	1·4937	1·4935	1·4937	1·4936	—
	C	—	1·4938	—	1·4944	1·4941	1·4942	1·4941	—
	Na	—	1·4965	—	1·4973	1·4969	1·4969	1·4969	1·4950
	Tl	—	1·4994	—	1·5001	1·5000	1·4999	1·4999	—
	F	—	1·5030	—	1·5037	1·5037	1·5036	1·5035	—
	G	—	1·5087	—	1·5096	1·5091	1·5091	1·5091	—
β Vibrations parallel symmetry axis *b*.	Li	1·4967	1·4950	1·4957	1·4957	—	—	1·4958	—
	C	1·4974	1·4955	1·4962	1·4961	—	—	1·4963	1·4942
	Na	1·5001	1·4984	1·4989	1·4989	—	—	1·4991	1·4970
	Tl	1·5031	1·5013	1·5021	1·5021	—	—	1·5022	—
	F	1·5070	1·5050	1·5057	1·5056	—	—	1·5058	1·5039
	G	1·5127	1·5107	1·5110	1·5113	—	—	1·5114	—
γ Vibrations parallel 1st median line.	Li	1·5108	—	1·5103	—	1·5101	1·5099	1·5103	—
	C	1·5114	—	1·5107	—	1·5106	1·5106	1·5108	—
	Na	1·5143	—	1·5137	—	1·5138	1·5139	1·5139	1·5120
	Tl	1·5178	—	1·5169	—	1·5170	1·5170	1·5172	—
	F	1·5216	—	1·5208	—	1·5208	1·5208	1·5210	—
	G	1·5272	—	1·5263	—	1·5266	1·5263	1·5266	—

The mean refractive index (mean of all three indices) for sodium light is 1·5033.

The values obtained by TOPSÖE and CHRISTIANSEN, which are given in the last column, and of which only the β values were obtained directly, do not show such close agreement with the author's values as was observed in the case of potassium zinc selenate.

The following general formula represents the intermediate refractive index of potassium magnesium selenate, with great accuracy, throughout the whole length of the spectrum. The index is corrected to a vacuum (by the addition of 0·0004 to the index as given in the table).

$$\beta = 1{\cdot}4852 + \frac{490\ 038}{\lambda^2} + \frac{222\ 100\ 000\ 000}{\lambda^4} \ldots$$

The α indices are closely reproduced by diminishing the constant 1·4852 by 0·0022, and the γ indices by increasing the constant by 0·0148.

Alteration of Refraction by Rise of Temperature.—A series of determinations carried out at 75° indicated that the indices diminish by about 0·0020 for 60° rise of temperature.

Axes of the Optical Ellipsoids.—Following are the calculated values of the axial ratios of the two optical ellipsoids :—

Axes of optical indicatrix :

$$\alpha : \beta : \gamma = 0{\cdot}9985 : 1 : 1{\cdot}0099.$$

Axes of optical velocity ellipsoid :

$$\mathfrak{a} : \mathfrak{b} : \mathfrak{c} = 1{\cdot}0015 : 1 : 0{\cdot}9902.$$

Molecular Optical Constants.—The values of these constants, calculated from the refractive indices with the aid of the density given on a preceding page, are as follows :—

Axis of optical indicatrix		α.	β.	γ.
Specific refraction, $\frac{n^2-1}{(n^2+2)d} = \mathfrak{n}$ . .	C	0·1232	0·1237	0·1267
	G	0·1264	0·1269	0·1300
Molecular refraction, $\frac{n^2-1}{n^2+2} \cdot \frac{M}{d} = \mathfrak{m}$.	C	61·07	61·30	62·81
	G	62·64	62·87	64·44
Specific dispersion, $\mathfrak{n}_G - \mathfrak{n}_C$		0·0032	0·0032	0·0033
Molecular dispersion, $\mathfrak{m}_G - \mathfrak{m}_C$		1·57	1·57	1·63
Molecular refraction, $\frac{n-1}{d}$ M . . .	C	103·63	104·09	107·13

Optic Axial Angle.—Following are the results of the measurements made with three excellent pairs of section plates, ground perpendicular to the first and second median lines.

Determination of Apparent Angle in Air of Potassium Magnesium Selenate.

Light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	61° 12′	61° 3′	61° 18′	61° 11′
C	61 11	61 2	61 17	61 10
Na	61 7	60 59	61 14	61 7
Tl	61 2	60 55	61 11	61 3
F	60 55	60 49	61 7	60 57

Determination of True Optic Axial Angle of Potassium Magnesium Selenate.

Light.	No. of section perp. 1st median line.	Observed values of 2H*a*.	No. of section perp. 2nd median line.	Observed values of 2H*o*.	Calculated values of 2V*a*.	Mean value of 2V*a*.
Li	1	35° 54′	1*a*	118° 17′	39° 30′	39° 43′
	2	35 52	2*a*	116 33	39 48	
	3	35 58	3*a*	116 45	39 52	
C	1	35 52	1*a*	118 12	39 29	39 42
	2	35 49	2*a*	116 29	39 46	
	3	35 56	3*a*	116 42	39 50	
Na	1	35 43	1*a*	117 41	39 26	39 38
	2	35 40	2*a*	116 0	39 42	
	3	35 44	3*a*	116 10	39 45	
Tl	1	35 32	1*a*	117 5	39 22	39 34
	2	35 28	2*a*	115 24	39 38	
	3	35 34	3*a*	115 30	39 42	
F	1	35 15	1*a*	116 20	39 14	39 25
	2	35 10	2*a*	114 40	39 29	
	3	35 14	3*a*	114 45	39 32	

TOPSÖE and CHRISTIANSEN obtained for the angle in air 62° 12′, and for the true angle 40° 22′, both referring to sodium light.

The *Dispersion of the Median Lines* was investigated by immersion in benzene, whose mean refractive index is approximately the same as that of the crystals. It proved to be exceedingly small, not exceeding 5′ between F and C, and while the largely preponderating number of determinations gave the indication that the first median line lies nearer to the morphological axis *a* for blue than for red, the amount is really so small as to lie within the limits of experimental error.

Effect of Rise of Temperature on the Optic Axial Angle.—Measurements at 80° showed that the angle in air increases 3° 10′ for 60° rise of temperature.

Rubidium Magnesium Selenate, $Rb_2Mg(SeO_4)_2,6H_2O$.

A determination of magnesium in 0·4133 gramme of the crystals employed afforded as result 0·0821 gramme magnesium pyrophosphate, which corresponds to 4·29 per cent. of magnesium. The theoretical percentage is 4·08.

Goniometry.

Twelve highly perfect small crystals were used in the goniometrical work. They were selected from the four most suitable crops.

Habit: thick tabular to prismatic.

Axial angle: $\beta = 74° 46'$.

Ratio of axes: $a : b : c = 0·7424 : 1 : 0·5011$.

Forms observed: $b = \{010\} \infty \text{P̶} \infty$; $c = \{001\} o\text{P}$; $p = \{110\} \infty \text{P}$; $q = \{011\} \text{P̶} \infty$; $o' = \{\bar{1}11\} + \text{P}$; $r' = \{\bar{2}01\} + 2\text{P̶} \infty$.

The accompanying table sets forth the results of the measurements.

The crystals of this salt are not distinguished by richness in the number of faces, and are frequently of a remarkably simple character. No other faces were present in a large proportion of the crystals examined besides those of the basal plane c, orthodome r', and clinodome q. The prevailing type was very similar to that of potassium magnesium selenate, except that the q faces were relatively more largely developed with respect to the faces of the basal plane, in accordance with the rule which has been established in the cases of the double sulphates and selenates already studied. No faces of the orthopinacoid a were observed, but those of the clinopinacoid b were frequently present, although small. The faces of the hemi-pyramid o' were but rarely observed, and were only measurable in one instance, the reflections in this case being good.

Morphological Angles of Rubidium Magnesium Selenate.

Angle measured.	No. of measurements.	Limits.	Mean observed.	Calculated.	Difference.
		° ′ ° ′	° ′	° ′	′
ac = 100 : 001 = β	—	—	—	74 46	—
as = 100 : 101	—	—	—	45 50	—
sc = 101 : 001	—	—	—	28 56	—
cr′ = 001 : $\bar{2}01$	14	63 25— 63 41	63 35	63 37	2
cs′ = 001 : $\bar{1}01$	—	—	—	38 20	—
s′r′ = $\bar{1}01$: $\bar{2}01$	—	—	—	25 17	—
r′a = $\bar{2}01$: $\bar{1}00$	—	—	—	41 37	—
r′c = $\bar{2}01$: $00\bar{1}$	13	116 19—116 32	116 25	116 23	2
ap = 100 : 110	—	—	—	35 38	—
pp′ = 110 : 120	—	—	—	19 28	—
p′b = 120 : 010	—	—	—	34 54	—
pb = 110 : 010	6	54 20— 54 26	54 23	54 22	1
pp = 110 : $1\bar{1}0$	20	71 4— 71 30	71 17	*	—
pp = 110 : $\bar{1}10$	20	108 27—108 54	108 43	108 43	0
cq = 001 : 011	54	25 39— 25 52	25 47	*	—
qb = 011 : 010	10	64 11— 64 14	64 13	64 13	0
qq = 011 : $01\bar{1}$	27	128 15—128 36	128 25	128 26	1
ao = 100 : 111	—	—	—	49 3	—
oq = 111 : 011	—	—	—	27 16	—
aq = 100 : 011	—	—	—	76 19	—
qo′ = 011 : $\bar{1}11$	—	—	—	34 34	—
o′a = $\bar{1}11$: $\bar{1}00$	—	—	—	69 7	—
co = 001 : 111	—	—	—	34 33	—
op = 111 : 110	—	—	—	43 7	—
cp = 001 : 110	38	77 34— 77 47	77 40	*	—
po′ = 110 : $11\bar{1}$	1	—	57 42	57 45	3
o′c = $11\bar{1}$: $00\bar{1}$	1	—	44 35	44 35	0
pc = 110 : $00\bar{1}$	38	102 1—102 30	102 20	102 20	0
bo = 010 : 111	—	—	—	70 14	—
os = 111 : 101	—	—	—	19 46	—
bo′ = 010 : $\bar{1}11$	—	—	—	65 15	—
o′s′ = $\bar{1}11$: $\bar{1}01$	—	—	—	24 45	—
sq = 101 : 011	—	—	—	38 0	—
qp = 011 : $\bar{1}10$	40	86 20— 86 41	86 30	86 29	1
ps = $\bar{1}10$: $\bar{1}0\bar{1}$	—	—	—	55 31	—
pq = $\bar{1}10$: $0\bar{1}\bar{1}$	40	93 21— 93 40	93 30	93 31	1
s′q = $\bar{1}01$: 011	—	—	—	45 4	—
qp = 011 : 110	39	63 21— 63 39	63 31	63 32	1
ps′ = 110 : $10\bar{1}$	—	—	—	71 24	—
pq = 110 : $0\bar{1}\bar{1}$	39	116 19—116 41	116 30	116 28	2
r′o′ = $\bar{2}01$: $\bar{1}11$	—	—	—	34 48	—
o′p = $\bar{1}11$: 110	—	—	—	92 37	—
pr′ = 110 : $20\bar{1}$	28	52 28— 52 48	52 37	52 35	2
r′p = $\bar{2}01$: 110	26	127 12—127 38	127 23	127 25	2

Total number of measurements, 474.

A typical crystal of this salt is represented later in fig. 2 (p. 274).

The common cleavage of the series is well marked in this salt, namely, parallel $r'\{\bar{2}01\}$.

Volume.

Relative Density.—The following results were obtained from four determinations with independent material :—

Weight of salt employed.	Sp. gr. at 20°/4°.	
6·4573	2·6803	Mean, 2·6805
6·1688	2·6808	
6·9637	2·6806	
6·7175	2·6803	

Molecular Volume.—$M/d = 588 \div 2{\cdot}6805 = 219{\cdot}36$.

Distance Ratios.—The following are the values of these ratios, obtained by combination of the above molecular volume with the axial ratios and axial angle already given :—

$$\chi : \psi : \omega = 6{\cdot}3001 : 8{\cdot}4861 : 4{\cdot}2524.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The plane of the optic axes (bi-normals) is the symmetry plane.

The sign of the double refraction is positive.

The extinction angle was found to be as follows, with the aid of two excellent section plates ground parallel to the plane of symmetry :—

Section 1 . . 1° 3′; Section 2 . . 1° 18′; Mean . . 1° 10′.

The direction is in front of the normal, that is, nearer to the inclined axis a. This extinction direction is the second median line. The first median line lies in the acute morphological axial angle ac, and is inclined 1° 10′ to the axis a. The second median line is inclined 16° 24′ to the vertical axis c.

Refractive Indices.—Six prisms, ground on separate crystals derived from four different crops, were employed in the determinations, which afforded the following results :—

Refractive Indices of Rubidium Magnesium Selenate.

	Light.	Prism 1.	Prism 2.	Prism 3.	Prism 4.	Prism 5.	Prism 6.	Mean.
α Vibrations parallel 2nd median line	Li	1·4975	—	—	1·4979	1·4981	1·4977	1·4978
	C	1·4981	—	—	1·4984	1·4985	1·4984	1·4983
	Na	1·5008	—	—	1·5011	1·5013	1·5011	1·5011
	Tl	1·5039	—	—	1·5041	1·5043	1·5042	1·5041
	F	1·5076	—	—	1·5077	1·5079	1·5076	1·5077
	G	1·5131	—	—	1·5132	1·5135	1·5132	1·5133
β Vibrations parallel symmetry axis b	Li	1·4994	1·4997	1·4997	—	1·5000	—	1·4997
	C	1·4999	1·5002	1·5002	—	1·5003	—	1·5002
	Na	1·5029	1·5032	1·5031	—	1·5033	—	1·5031
	Tl	1·5059	1·5059	1·5061	—	1·5062	—	1·5060
	F	1·5097	1·5096	1·5102	—	1·5096	—	1·5098
	G	1·5151	1·5150	1·5155	—	1·5153	—	1·5152
γ Vibrations parallel 1st median line	Li	—	1·5098	1·5101	1·5100	—	1·5100	1·5100
	C	—	1·5103	1·5107	1·5105	—	1·5106	1·5105
	Na	—	1·5133	1·5135	1·5135	—	1·5137	1·5135
	Tl	—	1·5165	1·5168	1·5167	—	1·5167	1·5167
	F	—	1·5203	1·5207	1·5204	—	1·5204	1·5205
	G	—	1·5262	1·5266	1·5265	—	1·5263	1·5264

The mean refractive index (mean of all three indices) for sodium light is 1·5059.

The intermediate refractive index β is represented accurately to near F of the spectrum by the following formula :—

$$\beta = 1{\cdot}4857 + \frac{739\ 403}{\lambda^2} - \frac{4\ 215\ 900\ 000\ 000}{\lambda^4} + \dots$$

The α indices are nearly reproduced by the formula if the constant 1·4857 is diminished by 0·0019, and the γ indices if the constant is increased by 0·0105. The indices thus yielded by the formulæ are corrected for a vacuum, being 0·0004 higher than those given in the table, which are not so corrected.

Alteration of Refraction by Rise of Temperature.—Determinations of refractive index at 75° indicated that the indices become reduced by 0·0019 by a rise of 60° in temperature.

Axes of the Optical Ellipsoid.—These are as follows :—

Axes of optical indicatrix :

$$\alpha : \beta : \gamma = 0{\cdot}9987 : 1 : 1{\cdot}0069.$$

Axes of optical velocity ellipsoid :

$$\mathfrak{a} : \mathfrak{b} : \mathfrak{c} = 1{\cdot}0013 : 1 : 0{\cdot}9931.$$

Molecular Optical Constants.—The calculated values of these constants are the following :—

Axis of optical indicatrix		α.	β.	γ.
Specific refraction, $\frac{n^2-1}{(n^2+2)d} = \mathfrak{n}$	C	0·1094	0·1098	0·1117
	G	0·1122	0·1125	0·1146
Molecular refraction, $\frac{n^2-1}{n^2+2} \cdot \frac{M}{d} = \mathfrak{m}$	C	64·33	64·54	65·66
	G	65·96	66·17	67·38
Specific dispersion, $\mathfrak{n}_G - \mathfrak{n}_C$		0·0028	0·0027	0·0029
Molecular dispersion, $\mathfrak{m}_G - \mathfrak{m}_C$		1·63	1·63	1·72
Molecular refraction, $\frac{n-1}{d} M$	C	109·31	109·72	111·99

Optic Axial Angle.—Three pairs of excellent section plates, ground parallel to the first and second median lines, afforded the following values :—

Apparent Angle in Air of Rubidium Magnesium Selenate.

Light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	74° 3′	74° 15′	74° 1′	74° 6′
C	73 58	74 9	73 52	74 0
Na	73 34	73 40	73 29	73 34
Tl	73 9	73 10	73 0	73 6
F	72 22	72 34	72 27	72 28

Determination of true Optic Axial Angle of Rubidium Magnesium Selenate.

Light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li	1	42° 55′	1*a*	112° 29′	47° 30′	
	2	42 50	2*a*	112 29	47 26	47° 26′
	3	42 47	3*a*	112 33	47 22	
C	1	42 49	1*a*	112 25	47 26	
	2	42 47	2*a*	112 26	47 24	47 24
	3	42 46	3*a*	112 32	47 21	
Na	1	42 18	1*a*	112 6	47 1	
	2	42 19	2*a*	112 6	47 2	47 3
	3	42 22	3*a*	112 4	47 6	
Tl	1	41 46	1*a*	111 40	46 37	
	2	41 49	2*a*	111 36	46 40	46 37
	3	41 44	3*a*	111 41	46 35	
F	1	41 0	1*a*	111 5	46 2	
	2	41 7	2*a*	111 1	46 9	46 6
	3	41 5	3*a*	111 7	46 6	

An examination of the optic figures in white light when the section perpendicular to the first median line is immersed in benzene, whose refraction is almost identical with that of this salt, shows the brushes fringed according to the following colour scheme :—

blue | red 1st M.L. red | blue.

The obtuse morphological axial angle *ac* is to the left in this scheme.

It is thus evident that the optic axial angle is greater for red than for blue, and determinations for C-light and F-light gave the angles for these respective wave-lengths 47° 34′ and 46° 3′, values so close to those derived from the determinations of 2Ha and 2Ho as seen in bromnaphthalene (*vide* the table) as to confirm the accuracy of the amount of dispersion of the axes shown in the table.

Dispersion of the Median Lines.—The actual circle readings obtained during the benzene immersion observations indicated that the first median line is so dispersed that it lies nearer to the morphological axis *a* for blue than for red by about 7′–8′ (between C- and F-light).

Effect of Rise of Temperature on the Optic Axial Angle.—Determinations of the apparent angle in air at 75° indicated that the 2E increases 25′ for 60° rise of temperature.

Cæsium Magnesium Selenate, $Cs_2Mg(SeO_4)_2,6H_2O$.

A determination of magnesium in 1·2312 gramme of crystals gave 0·2077 gramme of magnesium pyrophosphate, which corresponds to 3·64 per cent. of magnesium. The theoretical percentage of this metal is 3·51.

Goniometry.

Ten excellent crystals of this salt, selected from three of the best crops, were used in the goniometrical measurements.

Habit : flattened prismatic.

Axial angle : $\beta = 73° \, 43'$.

Ratio of axes : $a : b : c = 0·7314 : 1 : 0·4960$.

Forms observed : $b = \{010\} \infty \mathrm{P\!\!\!/} \infty$; $c = \{001\} o\mathrm{P}$; $p = \{110\} \infty \mathrm{P}$;
$q = \{011\} \mathrm{P\!\!\!/}\infty$; $o' = \{\bar{1}11\} + \mathrm{P}$; $r' = \{\bar{2}01\} + 2\mathrm{P\!\!\!/} \infty$.

The accompanying table represents the results of the measurements.

The cæsium salt of the magnesium group exhibits precisely the habitual configuration which has been shown to be characteristic of all the cæsium salts of the sulphate group of this series and of cæsium zinc selenate. It invariably presents large *q* (clinodome) faces, giving the prismatic appearance to the crystals, narrower (often very narrow) *c* basal plane faces, and for the end faces those of the primary prism *p* and of the hemi-pyramid *o′*, sometimes one and sometimes the other predominating. The clinopinacoid *b* is generally present as a strip, but the orthopinacoid *a* has not been observed. The faces of the orthodome *r′* were usually small, but in one of the best crops were frequently found much more prominently developed.

Morphological Angles of Cæsium Magnesium Selenate.

Angle measured.	No. of measurements.	Limits.	Mean observed.	Calculated.	Difference.
		° ′ ° ′	° ′	° ′	′
$ac = 100 : 001 = \beta$	—	—	—	73 43	—
$as = 100 : 101$	—	—	—	45 3	—
$sc = 101 : 001$	—	—	—	28 40	—
$cr' = 001 : \bar{2}01$	14	64 30— 64 47	64 39	64 31	8
$cs' = 001 : \bar{1}01$	—	—	—	38 46	—
$s'r' = \bar{1}01 : \bar{2}01$	—	—	—	25 45	—
$r'a = \bar{2}01 : \bar{1}00$	—	—	—	41 46	—
$r'c = \bar{2}01 : 00\bar{1}$	14	115 11—115 31	115 20	115 29	9
$ap = 100 : 110$	—	—	—	35 6	—
$pp' = 110 : 120$	—	—	—	19 28	—
$p'b = 120 : 010$	—	—	—	35 26	—
$pb = 110 : 010$	14	54 49— 55 2	54 54	54 54	0
$pp = 110 : 1\bar{1}0$	19	70 5— 70 19	70 12	*	—
$pp = 110 : \bar{1}10$	19	109 42—109 58	109 49	109 48	1
$cq = 001 : 011$	42	25 15— 25 34	25 29	*	—
$qb = 011 : 010$	14	64 23— 64 39	64 31	64 31	0
$qq = 011 : 01\bar{1}$	21	128 56—129 13	129 1	129 2	1
$ao = 100 : 111$	—	—	—	48 11	—
$oq = 111 : 011$	—	—	—	27 9	—
$aq = 100 : 011$	—	—	—	75 20	—
$qo' = 011 : \bar{1}11$	—	—	—	35 0	—
$o'a = \bar{1}11 : \bar{1}00$	—	—	—	69 40	—
$co = 001 : 111$	—	—	—	34 7	—
$op = 111 : 110$	—	—	—	42 37	—
$cp = 001 : 110$	39	76 36— 76 54	76 44	*	—
$po' = 110 : 11\bar{1}$	24	58 9— 58 23	58 16	58 24	8
$o'c = 11\bar{1} : 00\bar{1}$	24	44 51— 45 7	45 0	44 52	8
$pc = 110 : 00\bar{1}$	39	103 7—103 30	103 16	103 16	0
$bo = 010 : 111$	—	—	—	70 39	—
$os = 111 : 101$	—	—	—	19 21	—
$bo' = 010 : \bar{1}11$	20	65 13— 65 31	65 22	65 22	0
$o's' = \bar{1}11 : \bar{1}01$	—	—	—	24 38	—
$sq = 101 : 011$	—	—	—	37 37	—
$qp = 011 : \bar{1}10$	39	87 34— 87 45	87 40	87 42	2
$ps = \bar{1}10 : \bar{1}0\bar{1}$	—	—	—	54 41	—
$pq = \bar{1}10 : 0\bar{1}\bar{1}$	39	92 9— 92 28	92 20	92 18	2
$s'q = \bar{1}01 : 011$	—	—	—	45 16	—
$qp = 011 : 110$	40	62 55— 63 8	63 1	62 58	3
$ps' = 110 : 10\bar{1}$	—	—	—	71 46	—
$pq = 110 : 0\bar{1}\bar{1}$	40	116 47—117 6	116 59	117 2	3
$r'o' = \bar{2}01 : \bar{1}11$	20	34 53— 35 8	35 1	35 3	2
$o'p = \bar{1}11 : 110$	23	92 31— 92 56	92 43	92 34	9
$pr' = 110 : 20\bar{1}$	23	52 12— 52 22	52 16	52 23	7
$r'p = \bar{2}01 : 110$	23	127 37—127 54	127 44	127 37	7

Total number of measurements, 550.

Fig. 4 in the double sulphate memoir, representing cæsium magnesium sulphate, is equally applicable to the commonest type of cæsium magnesium selenate. It is reproduced in fig. 3 (p. 274) for the purpose of the comparison of the habits of the three magnesium selenates.

There is a good cleavage parallel r' $\{\bar{2}01\}$.

Volume.

Relative Density.—The following four determinations with separate quantities of material were made :—

Weight of salt employed.	Sp. gr. at 20°/4°.	
5·8081	2·9385	
5·7961	2·9391	Mean, 2·9388
6·2659	2·9389	
5·9848	2·9387	

Molecular Volume.—$M/d = 683 \div 2{\cdot}9388 = 232{\cdot}41$.

Distance Ratios.—The following distance ratios are afforded by combination of the molecular volume with the axial angle and ratios already given :—

$$\chi : \psi : \omega = 6{\cdot}3918 : 8{\cdot}7390 : 4{\cdot}3345.$$

Optics.

Cæsium magnesium selenate exhibits extraordinary optical characters, including crossed axial plane dispersion of the optic axes and great sensitiveness of the optic axial angle to change of temperature, together with corresponding apparently abnormal refraction phenomena. In this respect it is surprisingly similar to cæsium magnesium sulphate, the change of selenium for sulphur simply advancing all the optical constants without materially altering their mutual relations. Moreover, the abnormalities will be shown to be the direct result of the progression in optical properties which has so far throughout both sulphate and selenate series been found to accompany progress in the atomic weight of the alkali metal.

Orientation of Axes of Optical Ellipsoid.—The extinction angle in the symmetry plane, with respect to the normal to the basal plane, was determined with the aid of two sections parallel to the symmetry plane as usual, and afforded the following results :—

Section 1, 20° 35′; Section 2, 20° 25′. Mean, 20° 30′.

The direction is in front of the normal to (001), that is, nearer to the morphological axis a. This direction is the second median line for all wave-lengths of light from the red end of the spectrum as far as wave-length 466 in the blue ; beyond this the symmetry axis b becomes the second median line. The first median line lies in the symmetry plane for all wave-lengths, in the acute angle of the morphological axes ac, and is inclined to the axis a 20° 30′. The second median line so long as it remains in the symmetry plane lies in the obtuse angle of the axes ac, and for sodium light (to which the measurements of extinction refer) is inclined 36° 47′ to the vertical axis c.

The *double refraction* is of positive sign.

Refractive Indices.—These were determined with six excellent prisms, ground on six different crystals selected from three different crops. Each of the two prisms prepared to furnish α and β, and whose refracting edge was in each case parallel to the second median line (for wave-lengths as far as 466) and whose bisecting plane was that containing this edge and the symmetry axis, appeared to exhibit only one image of the Websky slit of the refractometer instead of the usual two; and this image had the further peculiarity of remaining permanent at all positions of the nicol, and for all wave-lengths, although its position naturally varied with the wave-length. On using the high-power eyepiece, the single image was clearly resolved into two images separated by 1′ for red light, approximating again to a single image as the greenish-blue was approached. One of the two images corresponded to α, and extinguished with the nicol at 90°; the other, corresponding to β, extinguished when the nicol was rotated to 0°. For F-light the positions of the two images were identical, and for G-light the two images were found to have passed each other, the one which was formerly right being now left and *vice versâ*, the amount of separation being about 1°. Hence for G-light the directions in the crystal parallel to which the α and β vibrations occur are interchanged, compared with those corresponding to wave-lengths on the red side of the crossing wave-length.

The actual results of the refractive index determinations are set forth in the accompanying table. It will be observed that the difference between α and β works out to 0·0002 for Li-light. Considering the minuteness of this quantity, it is as well to have an independent check upon its accuracy, and this is afforded by calculation from the optic axial angle for this wave-length, assuming also the accuracy of one of the two indices, either α or β, and of course of γ, with the aid of the formula

$$\text{Cos Va} = \sqrt{\left(\frac{1/\beta^2 - 1/\gamma^2}{1/\alpha^2 - 1/\gamma^2}\right)}.$$

The value of α thus obtained is 1·51433, or 0·00017 less than β_{Li}, a value for the difference which satisfactorily confirms that (0·0002) given in the table as the result of the determinations.

The wave-length for which the indices α and β are truly identical is 466 in the blue, as will subsequently be shown in considering the optic axial angle.

Refractive Indices of Cæsium Magnesium Selenate.

Index.	Direction of vibrations.	Light.	Prism 1.	Prism 2.	Prism 3.	Prism 4.	Prism 5.	Prism 6.	Mean.
α	Parallel 2nd median line lying in symmetry plane.	Li	1·5142	1·5143	1·5144	1·5143	1·5143	—	1·5143
		C	1·5147	1·5148	1·5149	1·5148	1·5148	—	1·5148
		Na	1·5177	1·5179	1·5179	1·5179	1·5177	—	1·5178
		Tl	1·5209	1·5212	1·5211	1·5210	1·5209	—	1·5210
		F	1·5246	1·5250	1·5246	1·5250	1·5247	—	1·5248
	Parallel symmetry axis, b, now 2nd median line.	G	1·5302	—	—	—	1·5305	1·5306	1·5304
β	Parallel symmetry axis, b.	Li	1·5144	—	—	—	1·5145	1·5146	1·5145
		C	1·5149	—	—	—	1·5150	1·5151	1·5150
		Na	1·5178	—	—	—	1·5178	1·5181	1·5179
		Tl	1·5211	—	—	—	1·5210	1·5211	1·5211
		F	1·5247	—	—	—	1·5247	1·5249	1·5248
	In symmetry plane at right angles to 1st median line.	G	1·5303	1·5305	1·5303	1·5308	1·5306	—	1·5305
γ	Parallel 1st median line.	Li	—	1·5200	1·5200	1·5201	—	1·5201	1·5201
		C	—	1·5205	1·5205	1·5206	—	1·5206	1·5206
		Na	—	1·5237	1·5235	1·5237	—	1·5236	1·5236
		Tl	—	1·5270	1·5267	1·5270	—	1·5269	1·5269
		F	—	1·5306	1·5307	1·5310	—	1·5307	1·5308
		G	—	1·5363	1·5363	1·5366	—	1·5363	1·5364

The mean refractive index (mean of all three indices) for sodium light is 1·5198.

The intermediate index β, corrected to a vacuum, is very accurately expressed throughout the whole length of the spectrum by the following formula :—

$$\beta = 1{\cdot}5033 + \frac{522\ 267}{\lambda^2} - \frac{52\ 000\ 000\ 000}{\lambda^4} + \ldots$$

The α index is given by diminishing the constant 1·5033 by 0·0002 as far as C of the spectrum, by 0·0001 between C and Tl-light, it is afforded exactly by the formula for wave-lengths between that of Tl-light and that of blue light, while for G-light 0·0001 requires again to be subtracted from the constant. The γ index is afforded by the formula fairly accurately if the constant is increased by 0·0057.

Alteration of Refraction by Increase of Temperature.—Measurements were carried out at 70° in Na-light with prisms 3 and 5, affording respectively α and γ, and α and β. The latter prism exhibited only a single inextinguishable image of the slit at 70°, consisting of two identically situated (overlapping) images. It will be subsequently shown that at this temperature a uniaxial interference figure is also observed. Both prisms afforded values which indicated that the refractive indices are reduced by 0·0014 for 55° rise of temperature. For wave-lengths beyond the neighbourhood of wave-length 560 the α vibrations occur parallel to the symmetry axis b, and the β vibrations take place in the symmetry plane.

Ratio of Axes of Optical Ellipsoid.—These ratios are as follows :—

Axes of optical indicatrix :

$$\alpha : \beta : \gamma = 0{\cdot}9999 : 1 : 1{\cdot}0038.$$

Axes of optical velocity ellipsoid :

$$\mathfrak{a} : \mathfrak{b} : \mathfrak{c} = 1{\cdot}0001 : 1 : 0{\cdot}9963.$$

Molecular Optical Constants.—The calculated values of these constants are as under :—

Axis of optical indicatrix		α.	β.	γ.
Specific refraction, $\frac{n^2-1}{(n^2+2)d} = \mathfrak{n}$ . .	C	0·1026	0·1026	0·1035
	G	0·1052	0·1052	0·1062
Molecular refraction, $\frac{n^2-1}{n^2+2}\cdot\frac{M}{d} = \mathfrak{m}$.	C	70·06	70·08	70·72
	G	71·84	71·85	72·51
Specific dispersion, $\mathfrak{n}_G - \mathfrak{n}_C$		0·0026	0·0026	0·0027
Molecular dispersion, $\mathfrak{m}_G - \mathfrak{m}_C$		1·78	1·77	1·79
Molecular refraction, $\frac{n-1}{d}$ M . . .	C	119·64	119·69	120·99

Optic Axial Angle.—Three capital section-plates of this salt were obtained by grinding, out of large but very perfect crystals, perpendicular to the first median line. Such sections require to be very thick, 5 millims. or more, in order to exhibit clear interference figures, on account of the extraordinarily small amount of double refraction. Sections perpendicular to the second median line show no axial brushes even in bromnaphthalene, the obtuse angle of the optic axes being too large for measurement. The plan has accordingly been adopted which was employed in the case of cæsium magnesium sulphate, for the determination of the true optic axial angle, namely, measurement while the sections perpendicular to the first median line were immersed in a liquid whose refractive index was identical with the mean index of the crystal. Cedar oil answers the purpose admirably, its refractive index for sodium light being 1·520.

The optic axial interference figures afforded by thick section-plates are both exceptional and very beautiful. In white light a deeply coloured figure is presented, in which the hyperbolic brushes are broad spectra. In monochromatic light a rapidly changing series of figures is presented, commencing with the brushes separated by 30° for red light, and approximating closer and closer to each other through the yellow and green, until for F-light the separation is only 13°. Passing into the blue, the hyperbolic brushes eventually coalesce, for light of wave-length 466, into the uniaxial rectangular cross, and the lemniscates become circles. The exact wave-length was readily determined by taking the circle reading of the monochromatic illuminator corresponding to the production of the uniaxial figure, and ascertaining the wave-length to which this reading corresponds from the curve prepared in the calibration of the instrument. (*Vide* 'Phil. Trans.,' A, 1894, p. 925.) When the prism-circle of the illuminator is further rotated towards the violet, the brushes open out again, and separate along the vertical diameter of the field of the polariscope, the plane of the optic axes having crossed from the symmetry plane to a plane at right angles to the symmetry plane. These beautiful changes are illustrated by the first six photographic reproductions given in the accompanying Plate; they represent the interference figures observed at the ordinary temperature in Li-, Na-, Tl- and F-light, light of wave-length 466, and G-light respectively.

The actual measurements are recorded in the following two tables. The phenomena when the section-plate is immersed in the cedar-oil cell are precisely similar to those in air, the angles afforded being merely smaller, namely, the true angles of inclination of the optic axes within the crystal.

In both cases the angle for G-light was determined with the section rotated 90° in its own plane, so as to bring the new direction of separation horizontal, and therefore convenient for the measurement of the angle.

The determinations of the circle reading corresponding to uniaxiality were made repeatedly for each of the two positions of the section, and the mean of the whole is the value recorded in the table.

Determination of Apparent Angle in Air of Cæsium Magnesium Selenate.

Plane of optic axes.	Light.	Section 1.	Section 2.	Section 3.	Mean 2E.
In symmetry plane.	Li	30° 7′	31° 43′	30° 39′	30° 50′
	C	29 32	31 19	30 20	30 24
	Na	26 31	28 30	27 20	27 27
	Tl	21 4	23 33	22 11	22 16
	F	12 7	14 53	13 23	13 28
	Wave-length in blue	0 0	0 0	0 0	0 0
Perpendicular to symmetry plane.	G	21 13	18 8	18 59	19 27

Determination of True Optic Axial Angle of Cæsium Magnesium Selenate by Immersion in Cedar Oil.

Plane of optic axes.	Light.	Section 1.	Section 2.	Section 3.	Mean $2V_a$.
In symmetry plane.	Li	21° 4′	20° 53′	20° 31′	20° 49′
	C	20 49	20 39	20 14	20 34
	Na	18 47	18 36	18 22	18 35
	Tl	15 40	15 25	15 10	15 25
	F	10 38	10 23	10 29	10 30
	Wave-length in blue	0 0	0 0	0 0	0 0
Perpendicular to symmetry plane.	G	12 51	11 57	13 38	12 49

Circle Readings for production of Uniaxial Figure.

	Section 1.	Section 2.	Section 3.	Mean reading.
In air	4° 59′	5° 27′	5° 16′	5° 14′
In cedar oil . . .	5 23	5 27	5 18	5 23
		Mean of two series		5 19

The reading 5° 19′ corresponds to the passage of blue light of wave-length 466 through the exit slit of the monochromatic illuminator.

Dispersion of the Median Lines.—Although there is such large dispersion of the optic axes, the median lines remain fairly constant. The first median line lies nearer to the axis a for red C-light than for greenish-blue F-light by about 15′.

Effect of Rise of Temperature on the Optic Axial Angle.—Sections 1 and 3 were studied at temperatures up to 97°. The phenomena presented were highly interesting, for within this comparatively small range of temperature (the most that can be employed as regards the upward direction on account of the presence of water of crystallisation), the uniaxial cross and circles are produced for all wave-lengths of light in turn, from 466 in the blue to the extreme red of the spectrum. The axes are observed to begin to approach each other as soon as the temperature commences to rise appreciably.

The following table represents the temperatures (corrected for the slight conduction of the crystal holder as described in the memoir concerning cæsium selenate, 'Journ. Chem. Soc., Trans.,' 1897, 895) at which the uniaxial figure is produced for different wave-lengths of light :—

Mean Corrected Temperatures for Production of Cross.

For	F-light	at	34°
,,	Tl	,,	60
,,	Na	,,	78
,,	C	,,	91
,,	Li	,,	94

When the cross is produced for thallium light at 60° the axes are still separated 13° 0′ for sodium light and 20° 30′ for lithium light; and when the temperature attains 78° and the uniaxial figure is formed for sodium light, the axes remain separated 13° 30′ for lithium light.

Repeated heating of the same section would appear to slightly lower permanently the temperatures at which the cross is produced.

The second series of six figures in the Plate represent the phenomena observed at 78°, in Li-, C-, Na-, Tl-, and F-light, and in light of wave-length 466.

Comparison of the Three Magnesium Salts.

The *Morphological Angles* are compared in the accompanying table. The results deducible are as follows:—

The axial angle β of rubidium magnesium selenate is nearly the mean of the axial angles of potassium magnesium and cæsium magnesium selenates.

With only one exception all the morphological angles of the rubidium salt are intermediate in value between those of the potassium and cæsium salts. The exception, the angle bo', only escapes following the rule by 2′, an amount within the limits of experimental error.

The change in the exterior angles on passing from one salt to another of the triplet is not, as a rule, directly proportional to the change in the atomic weight of the alkali metal. The primary prism zone shows the greatest divergence from direct proportion, the change in ap being as 1 to 2.

Comparison of the Axial Ratios.

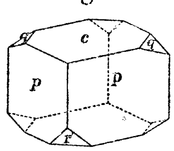
For potassium magnesium selenate . .	$a : b : c$ =	0·7485 : 1 : 0·5031
,, rubidium ,, ,, . . .	$a : b : c$ =	0·7424 : 1 : 0·5011
,, cæsium ,, ,, . . .	$a : b : c$ =	0·7314 : 1 : 0·4960

From this table it is clear that the morphological axial ratios of rubidium magnesium selenate are intermediate between those of potassium magnesium and cæsium magnesium selenates.

The general *Habit* of the crystals of the three salts exhibits very markedly the progression of type which has been established throughout the double sulphate series, and in the case of the zinc group of double selenates.

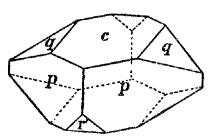
The potassium salt is characterised by a stout prismatic habit, the prism zone forming the prism, and by relatively large end basal plane faces. The cæsium salt is characterised by another prismatic habit, the prism faces being those of the clinodome $q\{011\}$, and the faces of the basal plane are generally reduced to a strip. The rubidium salt has been observed to exhibit every gradation between these two quite different habits, and a very large proportion of the crystals examined were of a clearly intermediate type, showing moderate-sized faces of the basal plane, and the prism and clinodome faces more or less equally developed. The three accompanying figures, representing typical crystals of the three salts, will render this progression of habit perfectly plain.

Fig. 1.



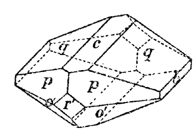
KMg selenate.

Fig. 2.



RbMg selenate.

Fig. 3.



CsMg selenate.

Comparison of the Morphological Angles of the Three Magnesium Salts.

Angle.	Potassium salt.		Difference.	Rubidium salt.		Difference.	Cæsium salt.	
	°	′	′	°	′	′	°	′
ac = 100 : 001 = β	75	42	−56	74	46	−63	73	43
as = 100 : 101	46	30	−40	45	50	−47	45	3
sc = 101 : 001	29	12	—	28	56	—	28	40
cr′ = 001 : $\bar{2}01$	62	53	+44	63	37	+54	64	31
cs′ = 001 : $\bar{1}01$	38	1	—	38	20	—	38	46
s′r′ = $\bar{1}01$: $\bar{2}01$	24	52	+25	25	17	+28	25	45
r′a = $\bar{2}01$: $\bar{1}00$	41	25	—	41	37	—	41	46
ap = 100 : 110	35	55	−17	35	38	−32	35	6
pp′ = 110 : 120	19	28	—	19	28	—	19	28
p′b = 120 : 010	34	37	+17	34	54	+32	35	26
pb = 110 : 010	54	5	—	54	22	—	54	54
cq = 001 : 011	26	0	−13	25	47	−18	25	29
qb = 011 : 010	64	0	—	64	13	—	64	31
ao = 100 : 111	49	43	−40	49	3	−52	48	11
oq = 111 : 011	27	27	−11	27	16	− 7	27	9
aq = 100 : 011	77	10	−51	76	19	−59	75	20
qo′ = 011 : $\bar{1}11$	34	16	+18	34	34	+26	35	0
o′a = $\bar{1}11$: $\bar{1}00$	68	34	+33	69	7	+33	69	40
co = 001 : 111	34	54	−21	34	33	−26	34	7
op = 111 : 110	43	34	−27	43	7	−30	42	37
cp = 001 : 110	78	28	−48	77	40	−56	76	44
po′ = 110 : $11\bar{1}$	57	13	+32	57	45	+39	58	24
o′c = $11\bar{1}$: $00\bar{1}$	44	19	+16	44	35	+17	44	52
bo = 010 : 111	69	57	+17	70	14	+25	70	39
os = 111 : 101	20	3	—	19	46	—	19	21
bo′ = 010 : $\bar{1}11$	65	16	− 1	65	15	+ 7	65	22
o′s′ = $\bar{1}11$: $\bar{1}01$	24	44	—	24	45	—	24	38
sq = 101 : 011	38	19	−19	38	0	−23	37	37
qp = 011 : $\bar{1}10$	85	34	+55	86	29	+73	87	42
ps = $\bar{1}10$: $\bar{1}0\bar{1}$	56	7	−36	55	31	−50	54	41
s′q = $\bar{1}01$: 011	44	55	+ 9	45	4	+12	45	16
qp = 011 : 110	64	5	−33	63	32	−34	62	58
ps′ = 110 : $10\bar{1}$	71	0	+24	71	24	+22	71	46
r′o′ = $\bar{2}01$: $\bar{1}11$	34	30	+18	34	48	+15	35	3
o′p = $\bar{1}11$: 110	92	54	−17	92	37	− 3	92	34
pr′ = 110 : $20\bar{1}$	52	36	− 1	52	35	−12	52	23

The *Cleavage Direction* of all three salts is identical, namely, parallel to the orthodome $r'\{\bar{2}01\}$, which is also common to the whole of the salts of the series yet studied.

Comparison of the Relative Densities.

Potassium magnesium selenate . .	2·3630	Diff. 0·3175
Rubidium ,, ,, . .	2·6805	Diff. 0·2583
Cæsium ,, ,, . .	2·9388	

The density increases with the atomic weight of the alkali metal, and is greater for the replacement of potassium by rubidium than for rubidium by cæsium in the proportion of 5 : 4. This proportion is the same as for the corresponding magnesium double sulphates, but the actual amounts of the differences are smaller; the difference between potassium magnesium sulphate and its rubidium analogue is 0·354, and between the latter and cæsium magnesium sulphate 0·288.

Comparison of the Molecular Volumes.

Potassium magnesium selenate . . .	209·73	Diff. 9·63
Rubidium ,, ,, . . .	219·36	Diff. 13·05
Cæsium ,, ,, . . .	232·41	

The molecular volumes show progression with the atomic weight of the alkali metal, but the replacement of rubidium by cæsium is accompanied by the greater change. The proportion is similar to that found for the double sulphates; the differences observed between the magnesium double sulphates were 9·33 and 13·19.

Comparison of the Distance Ratios.

	χ.	Diff.	ψ.	Diff.	ω.	Diff.
KMg selenate	6·2233	768	8·3144	1717	4·1829	695
RbMg ,, ...	6·3001	917	8·4861	2529	4·2524	821
CsMg ,, ...	6·3918		8·7390		4·3345	
		1685		4246		1516

The simplified ratios, taking ψ for KMg selenate as unity, are as follow:—

	χ.	Diff.	ψ.	Diff.	ω.	Diff.
KMg selenate	0·7485	92	1·0000	207	0·5031	83
RbMg ,, ...	0·7577	111	1·0207	304	0·5114	99
CsMg ,, ...	0·7688		1·0511		0·5213	
		203		511		182

These ratios (topic axes) indicate that there is an extension of the distance separating the structural units in all three axial directions, the maximum being along the symmetry axis and the minimum along the vertical axis. Also the change is greater for the replacement of rubidium by cæsium than for that of potassium by rubidium.

Comparison of Orientations of the Optical Indicatrix.

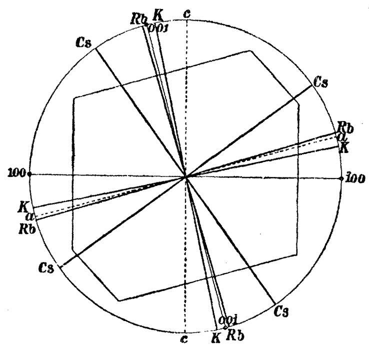
Inclination of Axis of Indicatrix to Vertical Axis *c*.

For potassium magnesium selenate . .	11°	18′	Diff. 5° 6′
,, rubidium ,, ,, . .	16	24	
,, cæsium ,, ,, . .	36	47	Diff. 20 23

The optical indicatrix rotates about the symmetry axis when one alkali metal is replaced by another, and by an amount which is very much greater when cæsium replaces rubidium than when rubidium replaces potassium.

This rotation of the optical ellipsoid is graphically represented in fig. 4, the dark lines representing the axes of the ellipsoid lying in the symmetry plane, namely, the first and second median lines.

Fig. 4.



The *Refractive Indices* are compared in the accompanying table.

It will be observed that the α and β indices of rubidium magnesium selenate lie between those of the other two salts. The γ indices of the potassium and rubidium salts are almost identical, and, indeed, those of the rubidium salt are slightly the lower. The reason for this apparent anomaly in the case of the γ indices is connected with a progressive change of dispersion, as will be shown under the next heading. That there is, however, a true progression of refraction is best exhibited by comparison of the values of the mean refractive index of each of the three salts, that is, the mean of the three indices of each salt for any one and the same wave-length of light.

Such mean indices for Na-light are compared in the table following that of the actual indices.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Comparative Table of Refractive Indices.

Index.	Light.	KMg selenate.	RbMg selenate.	CsMg selenate.
α	Li	1·4936	1·4978	1·5143
Vibrations	C	1·4941	1·4983	1·5148
parallel	Na	1·4969	1·5011	1·5178
second median	Tl	1·4999	1·5041	1·5210
line.	F	1·5035	1·5077	1·5248
	G	1·5091	1·5133	1·5304
β	Li	1·4958	1·4997	1·5145
Vibrations	C	1·4963	1·5002	1·5150
parallel	Na	1·4991	1·5031	1·5179
symmetry	Tl	1·5022	1·5060	1·5211
axis *b*.	F	1·5058	1·5098	1·5248
	G	1·5114	1·5152	1·5305*
γ	Li	1·5103	1·5100	1·5201
Vibrations	C	1·5108	1·5105	1·5206
parallel	Na	1·5139	1·5135	1·5236
first median	Tl	1·5172	1·5167	1·5269
line.	F	1·5210	1·5205	1·5308
	G	1·5266	1·5264	1·5364

Mean Refractive Indices of the Three Salts.

$\frac{1}{3}(\alpha + \beta + \gamma)$ for Na.

KMg selenate	1·5033	Diff. 26
RbMg ,, ...	1·5059	
CsMg ,, ...	1·5198	Diff. 139

The mean refractive indices of KMg, RbMg, and CsMg sulphates are respectively 1·4664, 1·4713, and 1·4877. The differences are 49 and 164, greater than in the selenate group.

The mean refractive index of rubidium magnesium selenate is thus seen to be intermediate between the mean indices of potassium and cæsium magnesium selenates; it lies, however, much nearer to that of the potassium salt than to that of cæsium magnesium selenate, as has been found general in the double sulphate series. The proportion of the differences for the two replacements is 1 : 5 for this group of double selenates, as against 1 : 4 for the double sulphates containing magnesium.

* The vibrations for G-light in the case of CsMg selenate occur in the symmetry plane.

Comparison of the Double Refraction, $Na_{\gamma-\alpha}$.

For KMg selenate	0·0170	Diff. 46
„ RbMg „	0·0124	
„ CsMg „	0·0058	Diff. 66

The double refraction is shown by the above table to decrease at an accelerating rate as the atomic weight of the alkali metal is increased. The amount of diminution for the first replacement is nearly double as much as the increase of mean refraction, which accounts for the fact already alluded to that the γ indices of rubidium magnesium selenate are slightly lower than those of potassium magnesium selenate. This latter apparent anomaly, therefore, is the direct outcome of the general rule so far established for the series as regards progressive diminution of double refraction.

The instructive axial ratios of the optical ellipsoid (calculated for Na-light) are compared in the following tables, and they are also graphically expressed in the curves of fig. 5. The table representing the values when β_{KMg} is taken as unity, and the series of dotted curves corresponding, exhibit the total change of the ellipsoid on passing from one salt to another, as distinguished from the change in the relations of the ellipsoidal axes of any one salt.

Comparison of the Optical Ellipsoids.

Optical Indicatrix.

	α.		β.		γ.	Double refraction.
KMg selenate	0·9985	:	1	:	1·0099	114
RbMg selenate	0·9987	:	1	:	1·0069	82
CsMg selenate	0·9999	:	1	:	1·0038	39

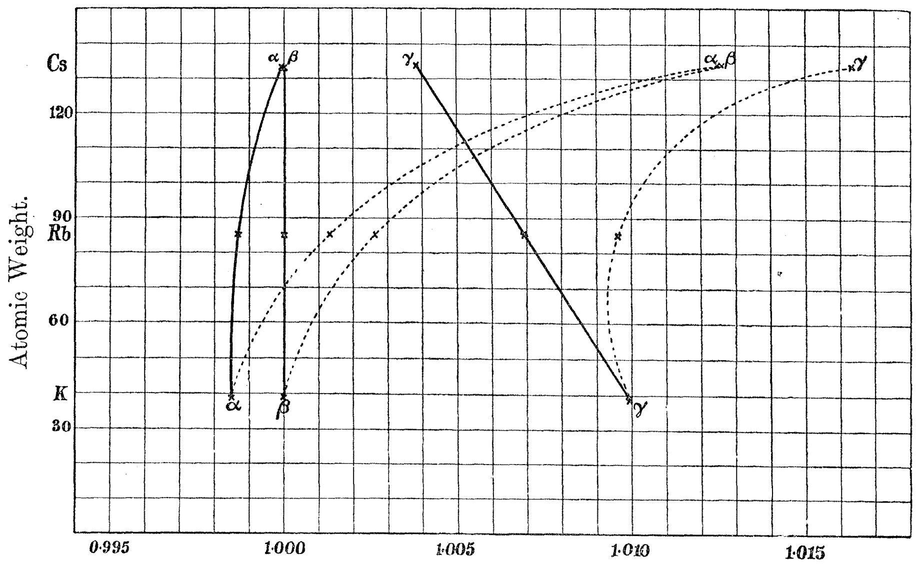
Optical Velocity Ellipsoid.

	$\mathfrak{a}$.		$\mathfrak{b}$.		$\mathfrak{c}$.	
KMg selenate	1·0015	:	1	:	0·9902	113
RbMg selenate	1·0013	:	1	:	0·9931	82
CsMg selenate	1·0001	:	1	:	0·9963	38

Indicatrix when $\beta_{KMg} = 1$.

	α.			β.			γ.	
KMg selenate .	0·9985		:	1		:	1·0099	
		28			27			3
RbMg selenate. . . .	1·0013		:	1·0027		:	1·0096	
		112			99			67
CsMg selenate	1·0125		:	1·0126		:	1·0163	

Fig. 5.



Axial ratios of optical indicatrix.

The axial ratios of the optical indicatrix of rubidium magnesium selenate are intermediate between those corresponding to the potassium and cæsium magnesium salts. The total change on passing from one salt to another is much the greater for the passage from the rubidium to the cæsium salt than for the passage from the potassium to the rubidium salt. The apparent anomaly as regards the latter change in the case of the γ ratio has already been fully explained. As regards total change, it will also be observed that the amount is considerably less along the direction of the axis γ than along the other two axes, along which the change is about equal.

The most striking circumstance about the magnesium series of curves, and in which they differ remarkably from those given in the author's last communication on the zinc group of double selenates, is the closeness of the α curve to the β straight line; and that the convergence of the α and γ curves, which graphically represents the diminution of double refraction with increase of atomic weight of the alkali metal, results in consequence in actual contact (intersection) of the α and β curves. It is thus at once made clear that the extraordinary optical properties of cæsium magnesium selenate, involving apparent uniaxial refraction phenomena and interference figures in convergent polarised light, are the direct results of the operation of the rule now established, so far as the series has yet been studied, that the double refraction decreases at an accelerating rate with increase of the atomic weight of the alkali metal. The curves are drawn from the values for sodium light at the ordinary temperature, and, although on the scale employed contact appears to be just attained, this is not really so for sodium light, although it requires a high-power eyepiece to separate the two images of the slit of the spectrometer in Na-light afforded by a 60°-prism ground to yield α and β. It would be true, however, for sodium light at the temperature of 78°, and even for lithium light at 94°, while it is absolutely true at the ordinary temperature itself for blue light of wave-length 466. For G-light intersection has occurred, and the curve hitherto called α has passed to the γ side of the β straight line.

This unusual optical character of cæsium magnesium selenate is similar to a remarkable extent to that of cæsium magnesium sulphate (p. 475 of the double sulphate memoir*); in the case of the latter salt, the wave-length for uniaxiality is 450.

Comparison of the Molecular Optical Constants.

Specific Refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = \mathfrak{n}$.

	For ray C (Hα). α.	β.	γ.	For ray Hγ near G. α.	β.	γ.
KMg sel. . . .	0·1232	0·1237	0·1267	0·1264	0·1269	0·1300
	138	139	150	142	144	154
RbMg sel. . . .	0·1094	0·1098	0·1117	0·1122	0·1125	0·1146
	68	72	82	70	73	84
CsMg sel. . . .	0·1026	0·1026	0·1035	0·1052	0·1052	0 1062

Molecular Refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = \mathfrak{m}$.

	For ray C (Hα). α.	β.	γ.	For ray Hγ near G. α.	β.	γ.
KMg sel. . . .	61·07	61·30	62·81	62·64	62·87	64 44
	3·26	3·24	2·85	3·32	3·30	2·94
RbMg sel. . . .	64·33	64·54	65·66	65·96	66·17	67·38
	5·73	5·54	5·06	5·88	5·68	5·13
CsMg sel. . . .	70·06	70·08	70·72	71·84	71·85	72·51

Specific Dispersion, $\mathfrak{n}_G - \mathfrak{n}_C$.

	α.	β.	γ.
KMg sel.	0·0032	0·0032	0·0033
RbMg sel.	0·0028	0·0027	0·0029
CsMg sel.	0·0026	0·0026	0·0027

Molecular Dispersion, $\mathfrak{m}_G - \mathfrak{m}_C$.

	α.	β.	γ.
KMg sel.	1·57	1·57	1·63
RbMg sel.	1·63	1·63	1·72
CsMg sel.	1·78	1·77	1·79

* 'Journ. Chem. Soc., Trans.,' 1896.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Molecular Refraction (GLADSTONE), $\frac{n-1}{d}$ M.

	α.		β.		γ.	
KMg sel.	103·63		104·09		107·13	
		5·68		5·63		4·86
RbMg sel.	109·31		109·72		111·99	
		10·33		9·97		9·00
CsMg sel.	119·64		119·69		120·99	

The whole of the specific and molecular optical constants of rubidium magnesium selenate are intermediate between those of potassium magnesium selenate and of cæsium magnesium selenate. The molecular refraction increases considerably more when rubidium is replaced by cæsium than when potassium is replaced by rubidium, the proportion being as 5 : 3. The amount of change is less along the axis γ than along the two other directions, along which the amounts are nearly identical.

These results are independent of the temperature, for it has been shown that the refraction diminishes when the temperature is raised, and the density naturally varies in the same direction. They are also independent of the formula employed, whether that of LORENZ or of GLADSTONE and DALE.

On comparing the molecular refraction constants of the magnesium double selenates with those previously given by the author for the magnesium double sulphates, in order to arrive at the effect of replacing sulphur by selenium, it is found that this replacement is accompanied by an increase of 6·9 to 7·1 Lorenz units or 12·7 to 13·3 Gladstone units. Allowing for the presence of two atoms, this gives for the increase per atom an average of 3·5 Lorenz and 6·5 Gladstone units. Both values refer to light of the wave-length of the C hydrogen line.

The specific and molecular dispersions of the magnesium double selenates are higher than those of the magnesium double sulphates, as was also observed with regard to the two zinc groups.

Comparison of the Optic Axial Angles.—The magnitudes of the optic axial angles of the three magnesium double selenates are not strictly comparable, on account of the extraordinary phenomena presented by the cæsium salt, involving the crossing of the plane of the optic axes. It has already been shown, in discussing the refraction phenomena, that this is the result of the operation of the rule of progression of the double refraction with the atomic weight of the alkali metal.

The closeness of the similarity between the optical behaviour of this salt and cæsium magnesium sulphate, already alluded to as regards the refraction, is very apparent as regards the optic axial angle phenomena. Cæsium magnesium sulphate and selenate exhibit respectively an angle for lithium light in the plane of symmetry of 18° 10′ and 20° 49′. In both cases the angle diminishes with diminishing wave-length until the axes unite, for wave-lengths 450 and 466 respectively, in the centre of the field of the polariscope, to produce the uniaxial figure consisting of rectangular diagonal cross and circular rings. For still shorter wave-lengths the axial brushes separate in the vertical field of the instrument at right angles to the symmetry plane, and for G-light the separations are respectively 7° 0′ and 12° 49′. The total dispersion between Li and G amounts to 25° 10′ and 33° 38′ respectively, the selenate thus being distinguished by considerably greater dispersion. All these figures refer to the true angles within the crystals.

The *Effect of Rise of Temperature on the Optic Axial Angle* in the case of cæsium magnesium selenate is likewise remarkably similar to that on the corresponding sulphate. In both cases the optic axial angle for wave-lengths on the red side of the crossing wave-length rapidly contracts as the temperature is raised, until the uniaxial figure is produced for each wave-length towards the red in turn. The temperatures at which the uniaxial figure is produced for the different wave-lengths are slightly lower in the case of cæsium magnesium selenate than in that of the sulphate. The temperatures for the latter salt given in the memoir concerning the double sulphates (*loc. cit.*, p. 371) were the actual temperatures read off on the thermometers of the heating apparatus of the larger Fuess polariscopical goniometer. In order to render them strictly comparable with those given in this memoir for cæsium magnesium selenate, they require to be corrected for the slight conduction of the crystal holder, as has been done in the case of the latter salt. These corrected temperatures are set forth below, and alongside them are given for comparison those of the selenate :—

Temperatures at which the Uniaxial Figure is produced.

Light.	CsMg selenate.	CsMg sulphate.
F	34°	47°
Tl	60	66
Na	78	80
C	91	93
Li	94	96

It is thus a fact that within the life-range of temperature of these dissociable water-containing salts the uniaxial figure is produced for every wave-length of the visible spectrum, and for each wave-length there is a definite temperature at which these interesting monoclinic crystals simulate uniaxial symmetry as regards their optical properties while retaining morphologically their exterior monoclinic symmetry.

The interference figures of cæsium magnesium selenate given in the Plate opposite p. 272 are almost equally applicable to cæsium magnesium sulphate.

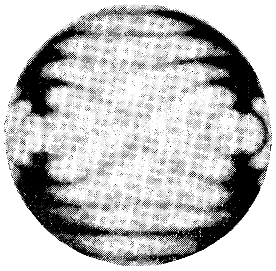
With regard to the effect of rise of temperature on potassium magnesium and rubidium magnesium selenates, it was observed that the optic axial angle of the former increased 3° 10′ for 60° of rise, and that the angle of the latter was but remarkably slightly affected by change of temperature, being increased by only 25′ for 60° of rise. Even in this magnesium group, therefore, in which the cæsium salt is abnormal, the character of progression which has been found to obtain throughout the double sulphate series, and in the zinc group of double selenates already studied, is still found to hold good, namely, a decided increase of the optic axial angle in the case of the potassium salt, a decided decrease in the case of the cæsium salt, and a comparative indifference to change of temperature in the case of the rubidium salt.

It is somewhat remarkable that four cases of large dispersion in crossed axial planes have been observed in the course of the author's work on the simple and double sulphates, namely, the rhombic sulphate of rubidium and selenate of cæsium and the monoclinic double sulphate and double selenate containing cæsium and magnesium. In each case the phenomenon has been rendered possible by the concurrence of two conditions, namely, extremely small double refraction (closeness of the α and γ indices of refraction) and the approximation to identity of the intermediate index of refraction β to either the α or γ index. The latter condition appears to be necessary for crossing of the axial planes to be possible, and the former condition for magnitude of separation of the optic axes in the two perpendicularly crossed planes. These two conditions are adequate to render the substance highly sensitive to slight differences in dispersion (that is, differences between α_{G-Li}, β_{G-Li}, and γ_{G-Li}), a dispersion difference of 0·0003 having been shown to be ample to cause reversal of the relations of the two nearly identical indices. Such substances are bound also to be highly sensitive to change of temperature, which usually provokes minute but influential changes in dispersion as well as producing different amounts of change of refraction along each of the three principal axes of the optical indicatrix.

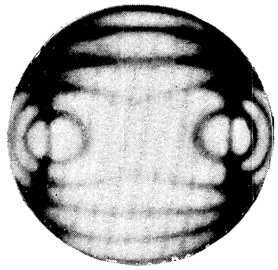
As the occurrence of these conditions has been general for all the four cases of crossed axial plane dispersion studied in detail by the author, it would appear probable that they afford a general explanation of this interesting phenomenon.

In the next communication the iron group of double selenates will be described.

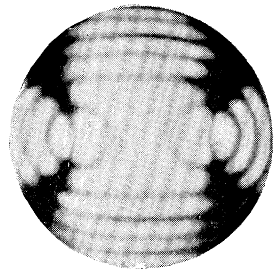
Series 1. *Ordinary Temperature.*



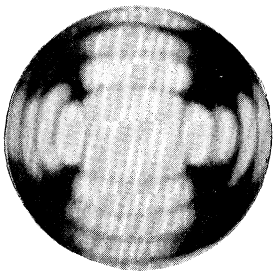
For Li-light.



For Na-light.



For Tl-light.



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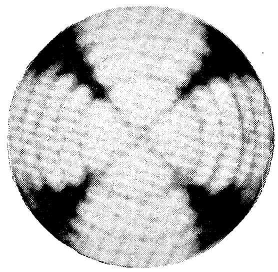
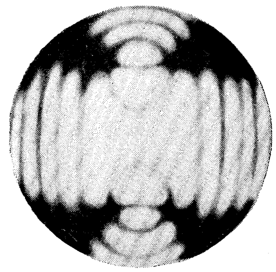
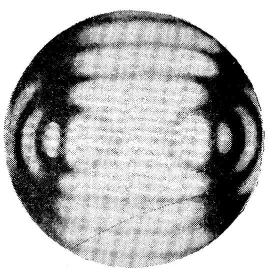


For light of wave-length 466.

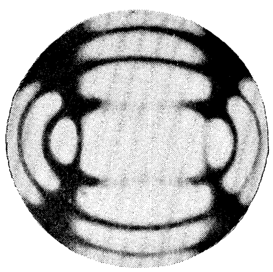


For G-light.

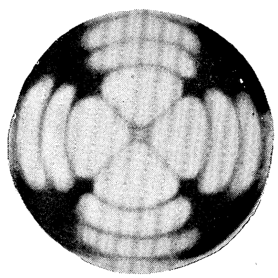
Series 2. *Temperature of* 78°.



For Li-light.



For C-light.



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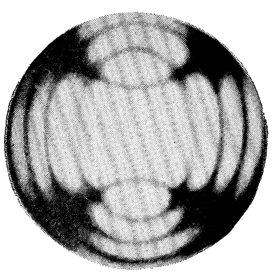

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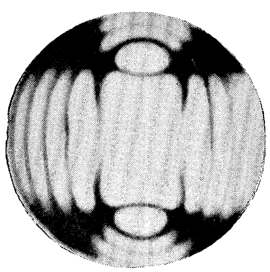

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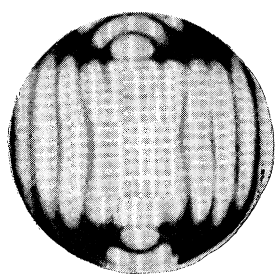

For light of wave-length 466.

Interference Figures afforded by Cæsium Magnesium Selenate in Convergent Polarised Light.

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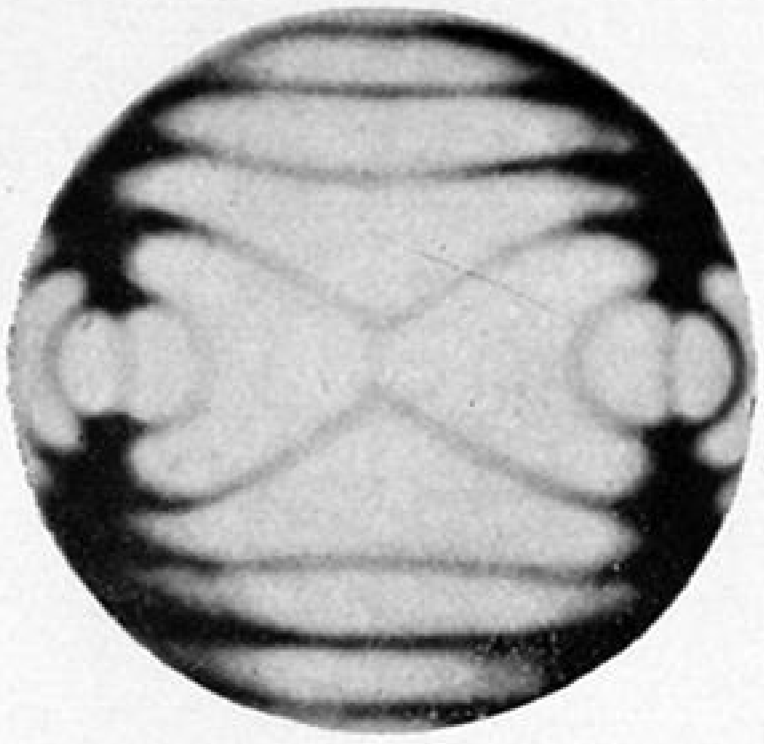
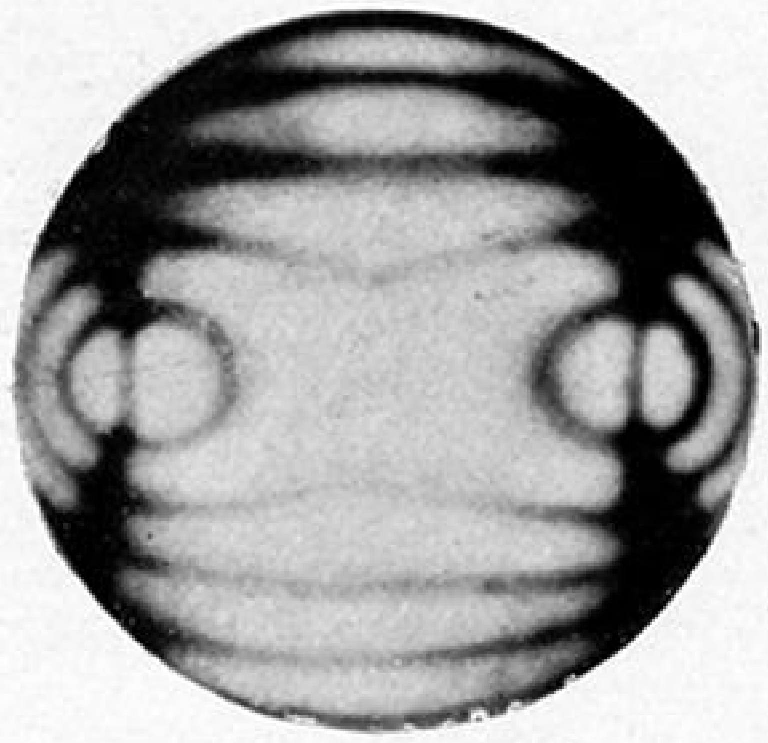
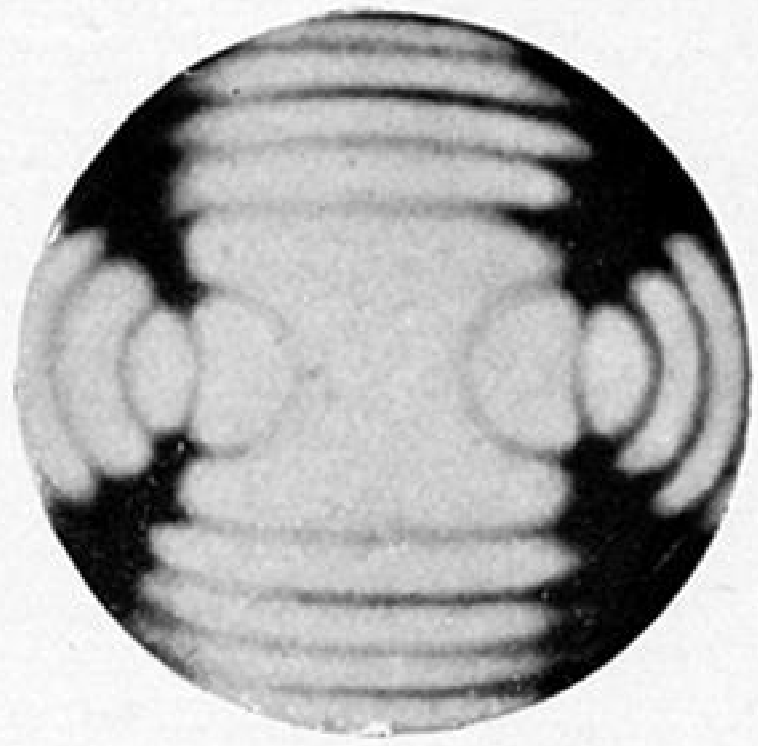
Series 1. *Ordinary Temperature.*



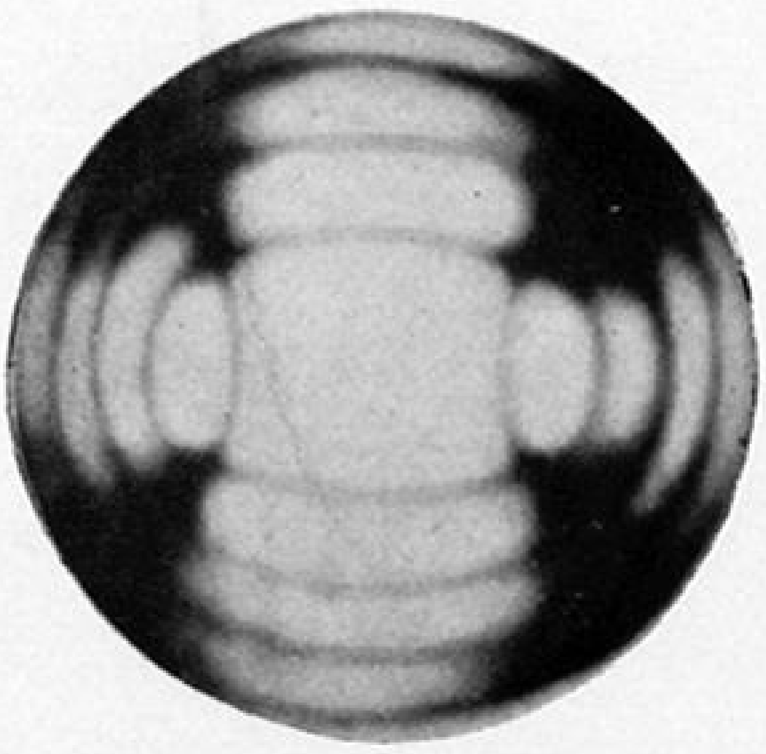
For Li-light.



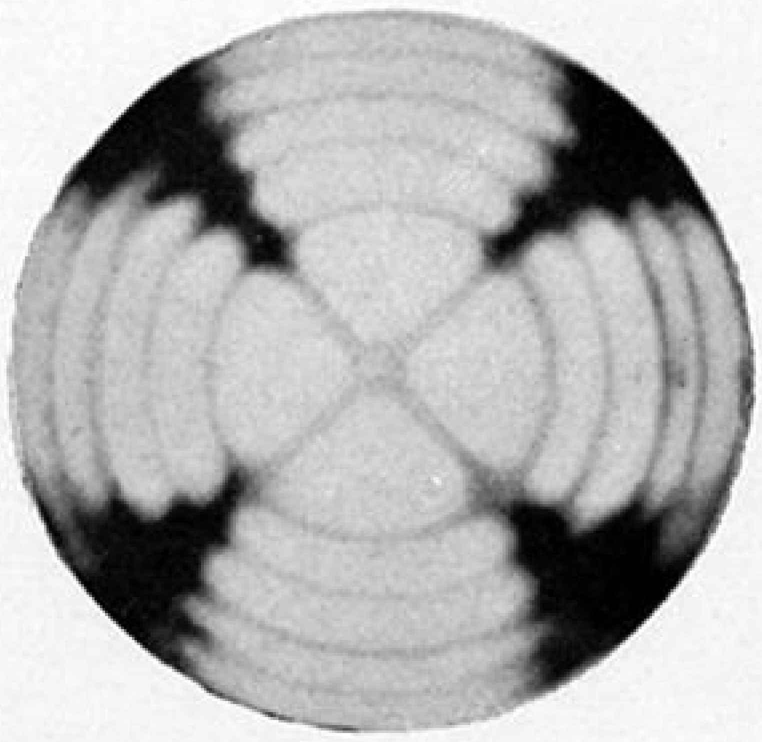
For Na-light.



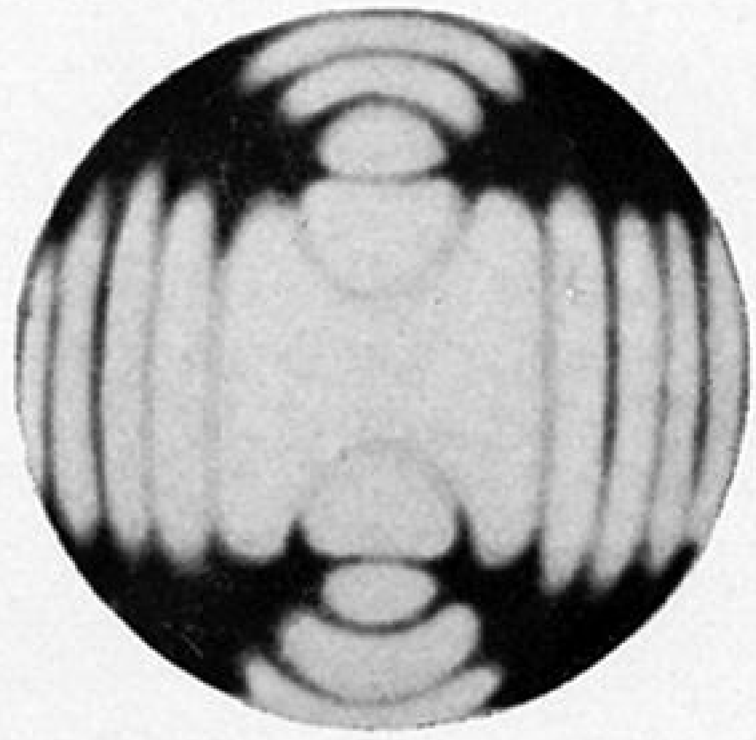
For Tl-light.



For F-light.

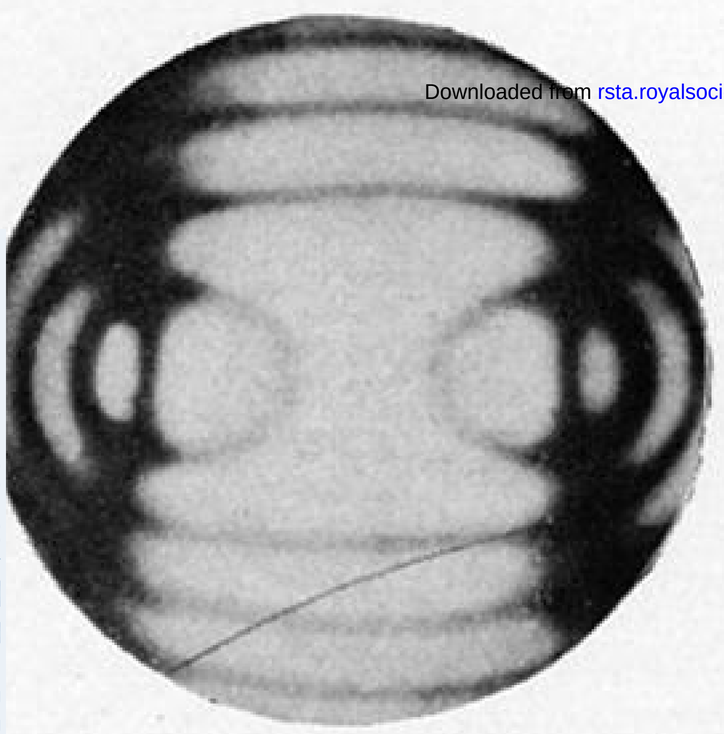


For light of wave-length 466.

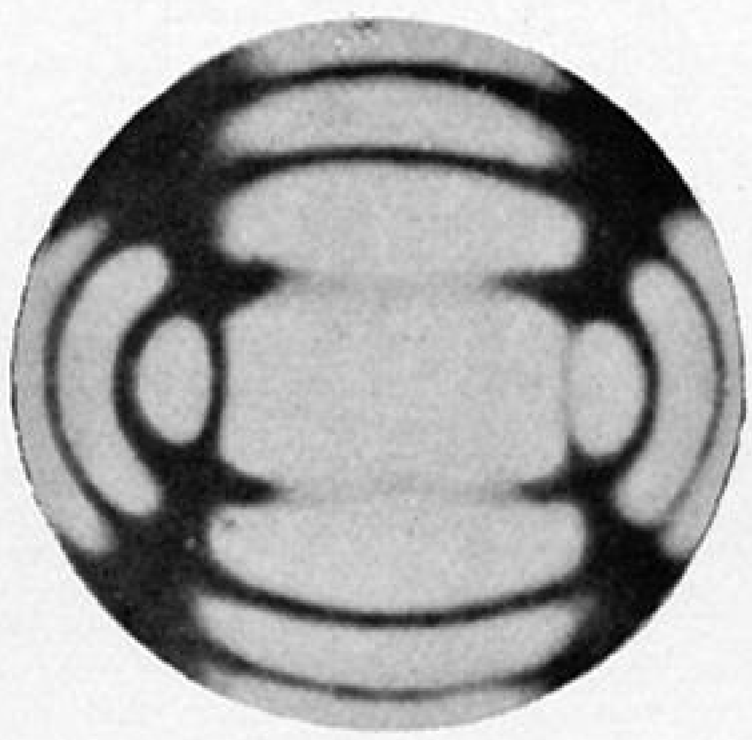


For G-light.

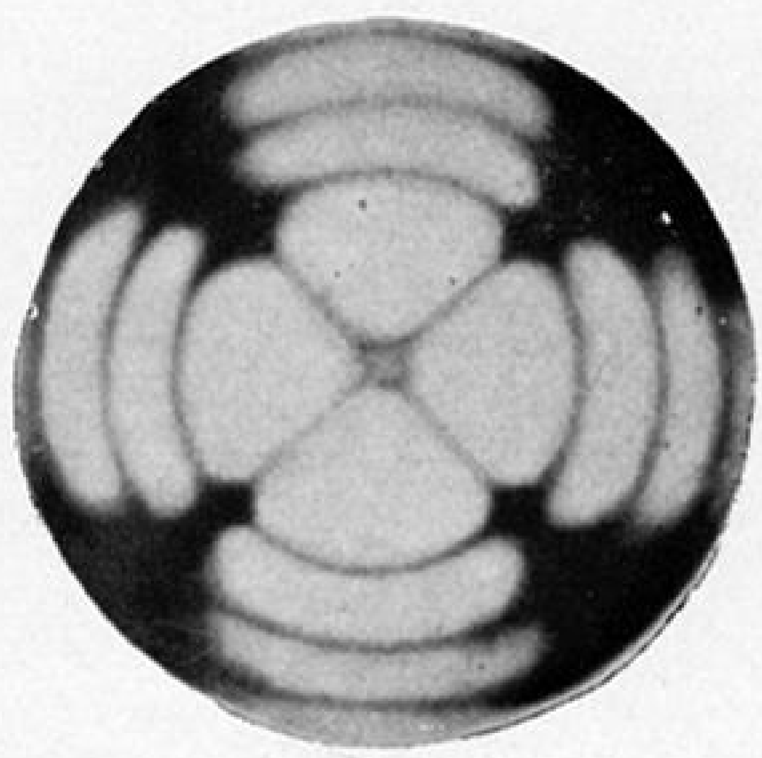
Series 2. *Temperature of* 78°.



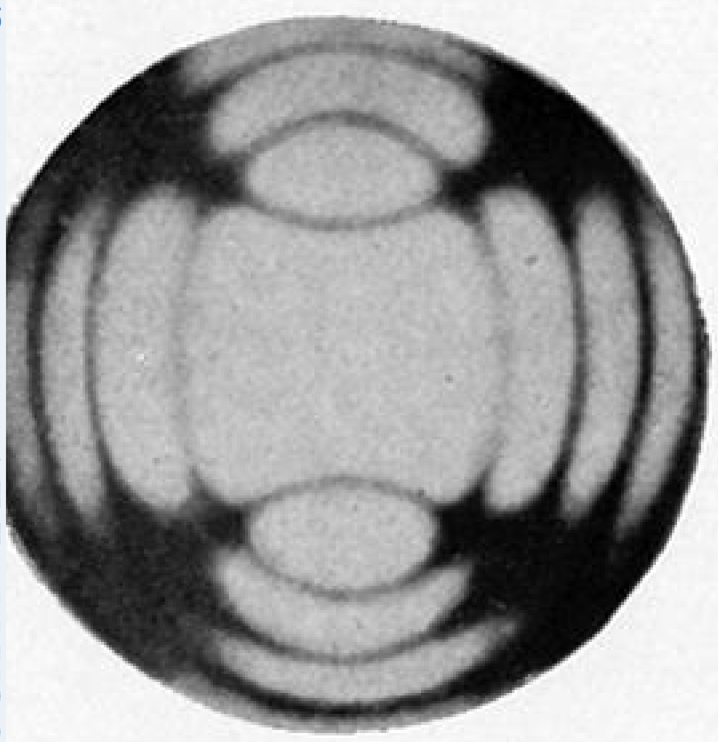
For Li-light.



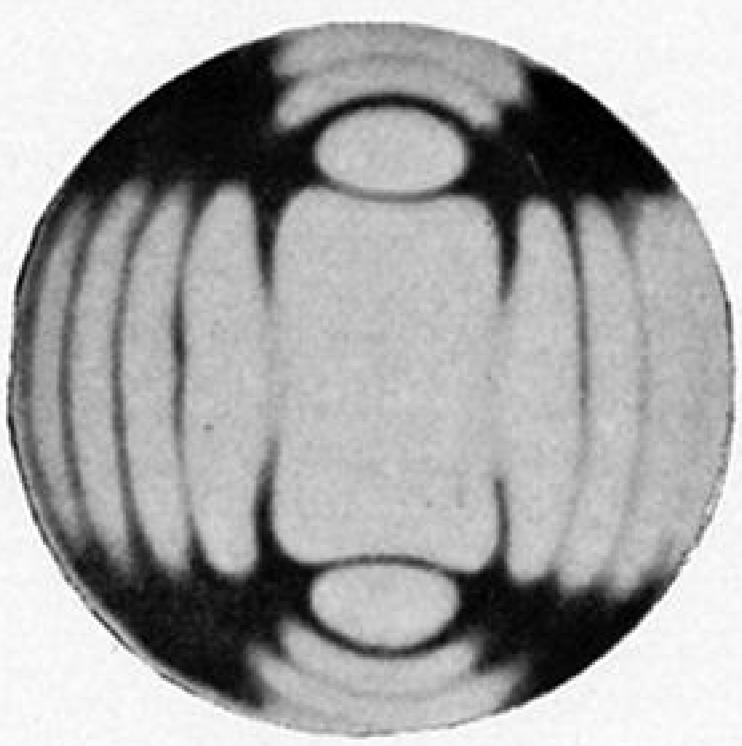
For C-light.



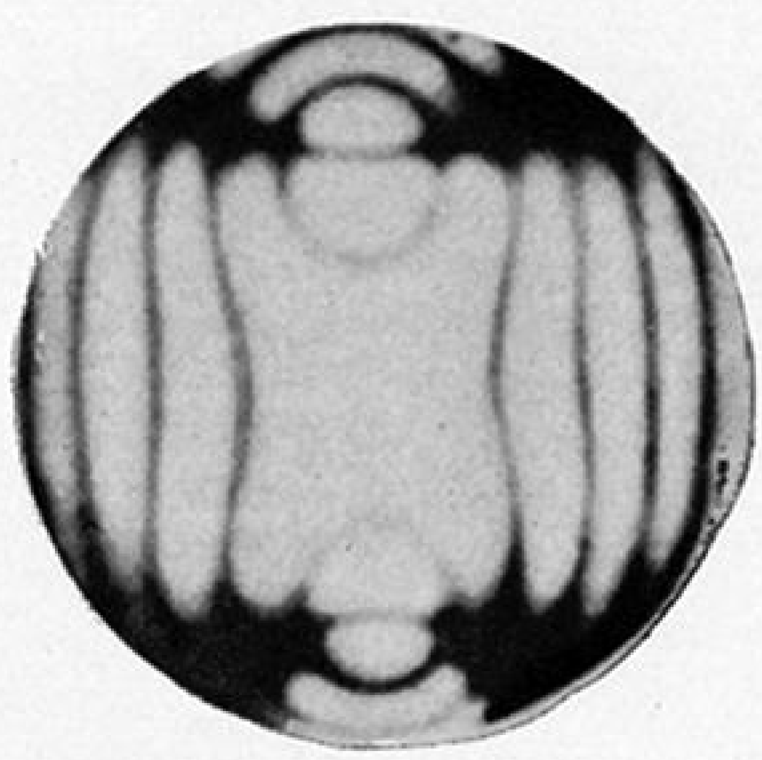
For Na-light.



For Tl-light.



For F-light.



For light of wave-length 466.

Interference Figures afforded by Cæsium Magnesium Selenate in Convergent Polarised Light.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES